Midterm 1 study guide

1.1 – Elementary operations

- systems of equations
  - solution set
  - inconsistent vs. consistent
- 2 equations in 2 variables: solution set is intersection of corresponding lines
- Parametric form of solution – parameter can be anything. (How to find a specific solution?)
- Augmented matrix
  - turn system into augmented matrix
  - 3 row operations allowed

1.2 – Gaussian Elimination

- row-echelon vs. reduced row echelon
- Gaussian algorithm: steps to get matrix in RE form.
- Gaussian elimination: steps to solve a system
- Rank = # of leading ones in RE form
- # of parameters needed in solution = (# of variables) – rank

1.3 – Homogeneous equations

- homogeneous equation: no constant terms
- homog. system: $\left[A \mid 0 \right]$

  0 vector

- trivial solution: all zeros, nontrivial: all other solutions
- # variables > # equations $\Longrightarrow$ nontrivial solutions
- linear combinations (definition?)
  - solutions of homog. systems are linear combinations of basic solutions (one for each parameter)

2.1 – Matrix notation + addition

- How to denote rows/columns/entries
- How to add matrices (when can you add them?)
- Transpose, symmetric matrices

2.2 – Matrix-vector multiplication

- When can we multiply A and $\vec{x}$?
- How to multiply? (whichever method works for you)
- Properties of mult + addition
- converting between system of eqns and matrix form $A\vec{x} = \vec{b}$.

- $A\vec{x} = \vec{b}$ vs. $A\vec{x} = \vec{0}$ (solution of $A\vec{x} = \vec{b}$ is $\underbrace{\vec{x}_1}_{\text{solution of } A\vec{x}=\vec{b}} + \underbrace{\vec{x}_0}_{\text{solution of } A\vec{x}=\vec{0}}$)

- Dot product

- Matrix transformations:
  - $T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ such that $T(\vec{x}) = A\vec{x}$.
    associated $m \times n$ matrix
  - T_A is transf. associated to A, defined $T_A(\vec{x}) = A\vec{x}$.

2.3 — matrix multiplication

- Application to transformations: $T_A \circ T_B = T_{AB}$
- When is AB defined? How to compute? (Dot product rule)
- Matrices don't usually commute!! $AB \neq BA$.
- Properties of mult + addition
- Block matrices (not super important)

2.4 — Matrix inverses

- Def of inverse/invertible
- inverses are unique, inverse of A is A^{-1}
- 2×2 determinant + inverse, $\det \neq 0 \Leftrightarrow$ invertible

- A invertible $\Rightarrow A\vec{x} = \vec{b}$ has unique solution $\vec{x} = A^{-1}\vec{b}$
- Can also use A^{-1} to solve things like $AB = C$ for B. $(\underbrace{A^{-1}AB}_{=B} = A^{-1}C)$
- Inversion method:

$$[A \quad I] \xrightarrow[\text{row ops}]{} [I \quad A^{-1}]$$

- know the properties of inverses (e.g. $(AB)^{-1} = B^{-1}A^{-1}$)
- equivalent conditions for being invertible.
- matrix transformation T_A has inverse $\Leftrightarrow$ A invertible. Then $(T_A)^{-1} = T_{A^{-1}}$.

2.5 – Elementary matrices

- Elementary matrix: $I \xrightarrow{\text{one row operation}} E$, 3 types
- If $A \to B$ by one row operation, then $B = EA$ (corresponding elem. matrix)
- How to find inverse of an elementary matrix (all elem. matrices are invertible)
- $\underset{\text{invertible}}{A} \to A_1 \to A_2 \to \cdots \to I$, with $A_1 = E_1A$, $A_2 = E_2E_1A$, $I = E_nE_{n-1}\cdots E_1A$

so $A^{-1} = E_nE_{n-1}\cdots E_1$ and $A = E_1^{-1}E_2^{-1}\cdots E_n^{-1}$

$\Rightarrow$ Invertible matrices are products of elem. matrices.